Conditional Excluded Middle in Informational Semantics*

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Abstract

Semantics for indicative conditionals (ICs) struggle with a problem inherited from the classical Stalnaker/Lewis debate on counterfactuals. On the one hand, ICs seem to satisfy Conditional Excluded Middle; on the other, ICs of the form $\phi > \neg\psi$ seem incompatible with *might*-conditionals of the form $\phi > \Diamond\psi$. These requirements are jointly unsatisfiable on standard notions of consequence. I show that a relative of Veltman's data and update semantics (1985, 1996), which I call *path semantics*, validates both. The analysis is confined to ICs, but can in principle be extended to counterfactuals.

1 Introduction

All theories of conditionals struggle with a tension between two plausible logical principles, which is inherited from the classical debate on counterfactuals between Stalnaker ([19]) and Lewis ([15]). On the one hand, conditionals seem to satisfy Conditional Excluded Middle, i.e. the principle that sentences of the form $(\phi > \psi) \vee (\phi > \neg\psi)$ are valid. On the other, conditionals of the form $\phi > \neg\psi$ seem incompatible with *might*-conditionals of the form $\phi > \Diamond\psi$. Unfortunately, these requirements are jointly unsatisfiable on a classical notion of consequence. As a result, most theories of conditionals drop one of them.

I show that the tension can be solved by moving to a new semantics that generates a nonclassical notion of consequence, which I call *path semantics*. Path semantics is a relative of informational semantics for epistemic modality in the style of Veltman ([20], [21]). In path semantics, all sentences are evaluated as true and false at sequences of information states. Conditionals have no quantificational force; rather, their antecedents are used to update the sequence of evaluation. Path semantics generates nonclassical notions of consequence that vindicate both the logical principles at stake.

I proceed as follows. §2 sets up the background problem; §3 briefly discusses solutions based on homogeneity; §4 introduces path semantics, and §5 discusses consequence. Given space constraints, throughout the paper I focus on epistemic conditionals and their *might*-counterparts, though both the puzzle and the account can be generalized to counterfactuals.

2 The puzzle

2.1 Conditional Excluded Middle

The first principle I consider is Conditional Excluded Middle (below), which I defend via two lines of argument.

Conditional Excluded Middle. (CEM) $\vDash (\phi > \psi) \vee (\phi > \neg\psi)$

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Argument #1: scopelessness. Epistemic conditionals with no overt modal appear to be scopeless with respect to logical operators: importing and exporting these operators inside and outside the consequent of a conditional makes no difference to truth conditions. For reasons of space here I only discuss negation, but the evidence for scopelessness includes the interactions between conditionals and quantifiers ([8], [5]), the adverb *only* ([4]), and comparative constructions ([10]).

Notice, first of all, that negation can be imported inside and outside the scope of a conditional without affecting truth conditions. The sentences in (1) are equivalent.

(1) a. It's not the case that, if Frida took the exam, she passed.
  b. If Frida took the exam, she didn't pass.

Notice also that the phenomenon persists with items that lexicalize negation, like *doubt* ($\approx$ *believe not*) and *fail* ($\approx$ *not pass*).

(2) a. I doubt that, if Frida took the exam, she passed.
  b. I believe that, if Frida took the exam, she failed.

The lack of semantic interaction with negative items is perfectly expected on a theory that vindicates CEM (assuming standard Excluded Middle in the background logic), but not on theories that treat conditionals as universal quantifiers.

Argument #2: probability. *Modulo* plausible assumptions, CEM is needed to vindicate basic probability judgments about conditionals. Both intuition and experimental results suggest that speakers judge that, at least for unembedded conditionals, the probability of a conditional equals the conditional probability of the consequent, given the antecedent.[1]

The Thesis. $Pr(\phi > \psi) = Pr(\psi \mid \phi)$
(for all ϕ, ψ, and for all Pr modeling rational credence)

Unfortunately, as has been shown repeatedly in the literature on so-called triviality results (see a.o. [14], [7]), the Thesis is untenable in full generality in truth-conditional frameworks. At the same time, one reasonable goal for semantic theory is to give a partial vindication of the Thesis. A plausible semantic theory should allow for assigning probabilities that conform to the Thesis to most simple conditionals in most ordinary contexts (for this claim, see e.g. [17]).

This piecemeal vindication seems highly desirable. Now, given some plausible assumptions, even this modest goal forces the adoption of CEM. Assume that, if a conditional $\phi > \psi$ conforms to the Thesis, then the corresponding conditional with a negated consequent $\phi > \neg\psi$ also conforms to the Thesis.[2] We can prove that, whenever $\phi > \psi$ has a consistent antecedent and conforms to the Thesis, the corresponding instance of CEM has probability 1.[3] In short:

[1] For a survey of classical experimental literature, see [3].

[2] Whatever one thinks about the Thesis in general, this assumption seems particularly intuitive, at least for simple conditionals. Even alleged counterexamples to the Thesis (see e.g. [9]) seem to conform to this assumption.

[3] Assuming the following uncontroversial principle, the proof is below.

Conditional noncontradiction. (CNC) $(\phi > \psi) \supset \neg(\phi > \neg\psi)$

i.	$Pr(\psi \mid \phi) + Pr(\neg\psi \mid \phi) = 1$	(probability calculus)
ii.	$Pr(\phi > \psi) + Pr(\phi > \neg\psi) = 1$	(i, Thesis)
iii.	$Pr((\phi > \psi) \wedge (\phi > \neg\psi)) = 0$	(CNC)
iv.	$Pr((\phi > \psi) \vee (\phi > \neg\psi)) = 1$	(ii, iii, probability calculus)

Fact. For all clauses ϕ, ψ (with ϕ consistent), and probability function Pr, such that $Pr(\phi > \psi) = Pr(\psi \mid \phi)$ and $Pr(\phi > \neg\psi) = Pr(\neg\psi \mid \phi)$, $Pr((\phi > \psi) \vee (\phi > \neg\psi)) = 1$.

Strictly speaking, Fact doesn't requre CEM; all we need is that a (large) number of instances of CEM are assigned probability 1. But a semantics that validates CEM immediately satisfies Fact. Conversely, accommodating Fact is a substantial challenge for any semantics that doesn't validate CEM.[4]

2.2 *If* and *might*

The second principle at stake states the incompatibility of $\phi > \neg\psi$ and $\phi > \Diamond\psi$.

If-Might Contradiction. (IMC) $(\phi > \neg\psi) \wedge (\phi > \Diamond\psi) \vDash \bot$

The evidence for IMC is straightforward. Discourses that involve conditionals of both forms are standardly heard as inconsistent; moreover, pairs of conditionals of this form can be used to generate disagreement. In addition, this infelicity persists also in linguistic environments that screen off pragmatic clashes, like supposition contexts (see [22]).

(3) # If Maria passed, Frida didn't pass; but, even if Maria passed, it might be that Frida passed.

(4) A: If Maria passed, Frida didn't pass.
B: I disagree. Even if Maria passed, it might be that Frida passed.

(5) # Suppose that, if Maria passed, Frida didn't pass, and that, if Maria passed, it might be that Frida passed.

Notice that IMC should be kept distinct from the following:

Duality. $\vDash (\phi > \Diamond\psi) \leftrightarrow (\neg(\phi > \neg\psi))$

Several classical frameworks (e.g., [15] [11], [12]) make IMC and Duality equivalent. But, as I show in §5, the two can come apart.

2.3 Collapse

Given a classical notion of consequence, CEM and IMC together entail the equivalence of $\phi > \psi$ and $\phi > \Diamond\psi$. The direction $\phi > \psi \vDash \phi > \Diamond\psi$ is uncontroversial; as for the other direction:

i.	$\phi > \Diamond\psi$	Assumption
ii.	$\quad\phi > \neg\psi$	Supposition for conditional proof
iii.	$\quad\phi > \neg\psi \wedge \phi > \Diamond\psi$	(i, ii, $\wedge$-Introduction)
iv.	$\quad\bot$	(iii, IMC)
v.	$\neg(\phi > \neg\psi)$	(ii-iv, Reductio)
vi.	$\phi > \neg\psi$	(v, CEM, Disjunctive syllogism)

Of course, this result is unacceptable. In response, classical theories drop one of CEM and IMC. Famously, Stalnaker ([19]) endorses CEM and rejects IMC, while most other theorists, ranging from Kratzer ([11] [12]) to Gillies ([6]) reject CEM. Both solutions are empirically costly, as is suggested by the discussion in this section.

[4]For example, on a Lewis/Kratzer-style semantics we need to make sure that, in each context, the ordering we use is appropriately tailored; this is bound to generate particularly implausible results in plenty of cases.

3 Homogeneity?

Some theorists ([4], [18], a.o.) try to capture both ICM and CEM by assuming that conditionals give rise to a so-called homogeneity inference. For concreteness I consider Schlenker's account, though my discussion generalizes to any homogeneity-based semantics. Before proceeding, a *caveat*. Both von Fintel and Schlenker treat homogeneity as a presupposition. Recently, Križ ([13]) has convincingly argued that homogeneity effects in natural language are not presuppositional. Here I follow Križ, though nothing I say depends on this.

Schlenker's account is based on an extended analogy between conditionals and plural definite descriptions. Roughly, conditionals are analyzed as modal descriptions of antecedent worlds. Here is an informal gloss of Schlenker's truth conditions:[5]

$\ulcorner \phi > \psi \urcorner$ is true at w iff the closest ϕ-worlds to w are ψ-worlds

Schlenker notices that this semantics vindicates a version of CEM when supplemented with homogeneity. It is well-known that plural descriptions like *the books* give rise to a homogeneity inference. Roughly, this is the requirement that sentences involving plural descriptions have a determinate truth value just in case all the atoms in the denotation of the description behave in the same way with respect to the predicate. For example, (6) is determinately true or determinately false just in case Mary either has read all the books or has read none.

(6) Mary read the books.

For current purposes, it's not important how homogeneity is triggered. What matters is that, if we pursue the analogy with descriptions, we expect conditionals to trigger a similar inference.

Homogeneity Inference (HI)
$\phi > \psi$ is true or false at w only if: either all closest ϕ-worlds to w are ψ-worlds, or all closest ϕ-worlds to w are $\neg\psi$-worlds

Given background assumptions about negation, HI allows us to vindicate a version of CEM.[6] *Modulo* a plausible semantics for *might*, IMC is also vindicated while avoiding collapse.[7]

While this is at first sight promising, there are reasons to reject the close analogy between plural descriptions and conditionals. Here I mention two.

Disanalogy #1: Probability. HI is of no help in vindicating intuitions about probability (see [1] for a similar argument about *will*). Consider:

(7) If Maria flipped the coin, the coin came up tails.

Suppose that you have no evidence one way or the other about how the coin landed. Plausibly, then, flip-and-heads-worlds and flip-and-tails-worlds are tied for closeness in your epistemic state.[8] In this situation, it seems that you should assign probability .5 to (7). Yet, if we assume

[5] On this gloss, as well as on Schlenker's theory, both conditionals and plural descriptions are taken to be nonmonotonic. As Schlenker makes clear, this is a non-essential feature of the theory and may be dropped.

[6] The background assumption is that negation has a Strong Kleene semantics, taking falsity into truth. Notice also that Schlenker's semantics, supplemented with HI, vindicates a weaker principle than CEM proper: $(\phi > \psi) \vee (\phi > \neg\psi)$ is valid *whenever* $\phi > \psi$ *is not undefined*

[7] Treat $\phi > \Diamond\psi$ as involving existential quantification over the closest ϕ-worlds. Then $\phi > \Diamond\psi$ is incompatible with $\phi > \neg\psi$, but the entailment to $\phi > \psi$ is blocked. To be sure, we have that, whenever $\phi > \Diamond\psi$ is true, $\phi > \neg\psi$ is not false; but it can be that $\phi > \Diamond\psi$ is true and $\phi > \neg\psi$ is undefined.

[8] I assume that the specific criteria we adopt for epistemic closeness won't matter here. If you think they do, just switch to a different example.

HI, (7) suffers from homogeneity failure and hence it is undefined. Now, it is unclear exactly what credence one should assign to statements with this status. But it seems both irrational and unusual to assign to them positive intermediate credence. For a comparison, ask yourself what credence you would assign to "Mary read the books" in a scenario where Mary read only half of the books.

This theoretical argument seems supported by recent experimental findings. Cremers, Križ and Chemla find ([2]) that subjects tend to follow different patterns when asked to assign probabilities, on the one hand, to sentences suffering from homogeneity failure and, on the other, to conditionals in contexts where not all possible antecedent worlds verify the consequent.

Disanalogy #2: Projection behavior. Homogeneity inferences project under attitude verbs. In particular, they display a projection behavior similar to that of presuppositions under attitude verbs or complex predicates that describe uncertainty, like *wonder*, or *be not certain*. For example, S *wonders whether* p, where p involves a homogeneity trigger, suggests the inference that S believes the relevant homogeneity claim.[9]

(8) a. Paula wonders whether the girls passed.
$\rightsquigarrow$ Paula believes: all girls passed $\vee$ all girls didn't pass
b. Everyone is not certain that the girls passed.
$\rightsquigarrow$ Everyone believes: all girls passed $\vee$ all girls didn't pass

Conversely, conditionals don't display a similar projection behavior.[10]

(9) a. Paula wonders whether, if Frida took the exam, she passed.
$\not\rightsquigarrow$ Paula believes: all her epistemic exam-worlds are pass-worlds $\vee$ all her epistemic exam-worlds are not-pass-worlds.
b. Everyone is not certain that Frida passed, if she took the exam.
$\not\rightsquigarrow$ Everyone believes: all their epistemic exam-worlds are pass-worlds $\vee$ all their epistemic exam-worlds are non-pass-worlds

(I'm assuming that conditionals in the scope of an attitude verb quantify over the subject's epistemic state. The problem could be reproduced on other accounts, e.g. accountswhere conditionals directly take their domain of quantification from attitude verbs, in the style of [22].)

4 Path semantics

The puzzle I outlined in §2 is due to the tension generated by three individually plausible but seemingly inconsistent principles.

Conditional Excluded Middle. (CEM)	$\vDash (\phi > \psi) \vee (\phi > \neg\psi)$
If-Might Contradiction. (IMC)	$(\phi > \neg\psi) \wedge (\phi > \Diamond\psi) \vDash \bot$
Nonfactivity of Might-Conditionals. (NMC)	$\phi > \Diamond\psi \nvDash \phi > \psi$

When framed in this way, the puzzle is clearly reminiscent of a classical puzzle about epistemic *might*. Yalcin ([22]) notices that sentences of the form $\neg\phi \wedge \Diamond\phi$ seem inconsistent:

(10) # It's not raining and it might be raining.

[9] For the claim that S *wonders whether* Q_p presupposes S *believes* p, see [16].

[10] I assume that conditionals in attitude reports quantify over epistemic worlds of the subject of the attitude. The point still goes through if we assume, loosely following Yalcin ([22]), that conditionals merely borrow the domain of quantification from the relevant attitude verbs.

Yet this intuition is hard to vindicate on a classical semantics for *might*, on which the following two principles are inconsistent.

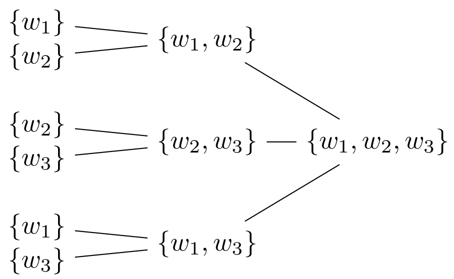
Epistemic Contradiction. $\neg\phi \wedge \Diamond\phi \vDash \bot$
Nonfactivity of Epistemic Modality. $\Diamond\phi \nvDash \phi$

The puzzle in §2 is then a generalization of Yalcin's puzzle (to see this, just instantiate $\top$ for 'ϕ' in our triad). Yalcin takes his puzzle as motivation for pursuing an informational semantics for epistemic modals, in the style of Veltman ([20], [21]). I pursue a similar goal. The new semantics will allow us to define a nonclassical notion of consequence, on which the three offending principles are consistent.

4.1 Paths

On standard informational semantics (see e.g. [20], [21]), sentences are evaluated as true or false relative to an *information state*, which here I model as a set of worlds. Conversely, the basic unit of path semantics is an *information path*. Informally, an information path is a sequence of information states that starts from the empty set and expands into a larger information state, adding one world at a time. More formally, we can define paths as follows. Let i be any information state. An information path (in i) is a sequence of subsets of i that is (i) ordered by subsethood and (ii) maximal, in the sense that there is no larger sequence of subsets of i that is ordered by subsethood. An information state uniquely determines a set of information paths; I use PATH(i) to denote the set of paths determined by information state i.

It is useful to model paths via branching diagrams. For an example, here is the set of paths determined by the information state $i = \{w_1, w_2, w_3\}$. (To avoid clutter, I leave out the empty set, which is the beginning point of each path.)



Intuitively, paths model possible ways in which information may grow. This becomes clearer by reading paths from right to left. A move to a smaller set in a path represents a possible transition from a less informed to a more informed state. (I will keep writing paths from the smallest to the largest set, left to right, because it makes the formalism more intuitive.)

4.2 Truth and falsity at a path

I state a semantics for a propositional language involving atomic sentences, Boolean connectives, epistemic modals, and conditionals. All sentences are evaluated relative to a path. This evaluation procedure is supplemented with a notion of update, which plays a key role in evaluating conditionals. In the next paragraphs, I state the semantics and go through some examples.

4.3 Semantics

I take an **information path in** i to be a maximal sequence of elements of $\wp(i)$, ordered by the subset relation. I use the customary square brackets '$[\![\cdot]\!]$' notation for the interpretation function and relativize interpretation to a path parameter P. I also assume a background model $\langle W, V\rangle$, with W a set of worlds and V a valuation function mapping atomic sentences to $\{0, 1\}$.

These are the clauses for atomic sentences, connectives, and modals.

Atoms: $[\![A]\!]^P = 1$ iff $w : \mathsf{min}(P) = \{w\}$, is s.t. $V(w, A) = 1$
($\mathsf{min}(P)$ is the smallest non-empty member of P)

$[\![\neg\phi]\!]^P = 1$ iff $[\![\phi]\!]^P = 0$

$[\![\phi \vee \psi]\!]^P = 1$ iff $[\![\phi]\!]^P = 1$ or $[\![\psi]\!]^P = 1$

$[\![\phi \wedge \psi]\!]^P = 1$ iff $[\![\phi]\!]^P = 1$ and $[\![\psi]\!]^P = 1$

$[\![\Diamond\phi]\!]^P = 1$ iff for some $w \in \bigcup P$, $V(w, \phi) = 1$

$[\![\Box\phi]\!]^P = 1$ iff for all $w \in \bigcup P$, $V(w, \phi) = 1$

To give a semantics for conditionals, we first need to define the update $P[\phi]$ of a path P with a formula ϕ. To do this, we define two preliminary notions. The first:

ϕ is **true throughout** an information state i iff, for all P in $\textsc{path}(i)$, $[\![\phi]\!]^P = 1$

I.e.: ϕ is true throughout an information state i just in case it is true at all the paths that are generated by i. Second, we define the notion of the update of an information state with ϕ.

i' **is the update of** i **with respect to** ϕ (in short: $i[\phi]$) iff:

i. $i' \subseteq i$;
ii. ϕ is true throughout i';
iii. there is no larger set that meets conditions (i) and (ii).

In short: the update of i with respect to ϕ is the largest subset of i such that ϕ is true at all the paths generated by it.[11] It is easy to check that this yields intuitive results.[12]

Finally, we define the update of P with respect to ϕ. This is just pointwise intersection of each information state in P with the updated information state that generates P:

Update of ϕ **with respect to** ϕ: $P[\phi] = P \Cap (\bigcup P)[\phi]$

(with: $P \Cap i = \langle p_1 \cap i, \ldots, p_n \cap i \ldots\rangle$)

At this point, we can define truth at a path for conditionals in terms of update.

$[\![\text{if } \phi, \psi]\!]^P = 1$ iff $[\![\psi]\!]^{P[\phi]} = 1$

[11] Given the sentences we're able to express in the language, there will always be a unique such set.

[12] Some examples: for any nonmodal sentence ϕ, $i[\phi]$ is the set of ϕ-worlds in i; $i[\Diamond\phi]$ is i itself if i contains a ϕ-world, and $\varnothing$ otherwise; $i[\neg\phi \wedge \Diamond\phi]$ is invariably $\varnothing$. These predictions are in line with update semantics ([21]). An interesting, and in my view welcome, divergence: $i[\Box\phi]$ is identical to $i[\phi]$, i.e. i updated with $\Box\phi$ is the set of ϕ-worlds in i.

4.4 Examples

It is useful to see how a few sentences are evaluated at a sample path. For illustration, consider: ('M' and 'F' stand for the propositions that Maria passed and that Frida passed):

(11) $\langle \varnothing, \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}\}, \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}\}, \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}}\}, \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}}, w_{\mathrm{MF}}\}\rangle$

Nonmodal sentences. Nonmodal sentences are invariably evaluated at the first non-empty set in a path, which by design contains a unique world. As a result, the semantics of nonmodal sentences is fully classical. Here are some examples of sentences that are true at (11).

(12) a. Maria didn't pass.
b. Maria passed or Frida didn't pass.
c. Neither Maria nor Frida passed.

Modalized sentences. $\Diamond\phi$ and $\Box\phi$ are evaluated at the information state that generates the path: technically, this means that they are evaluated at the union set of the path. As a result, the semantics of modalized claims reduces to standard informational semantics. For illustration, (11) makes true:

(13) a. It might be that Maria passed.
b. It might be that Frida didn't pass.
c. It might be that Maria didn't pass and Frida did.

Conditionals. Conditionals make full use of the path structure. Conditional antecedents update the path of evaluation; the consequent is evaluated at the updated path. Consider:

(14) If Maria passed, Frida passed.

We first use the antecedent *If Maria passed* to update the path. Starting from (11) and following the definition of update above, we get:

$$\begin{array}{ccccc}
\langle \varnothing, & \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}\}, & \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}\}, & \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}}\}, & \{w_{\bar{\mathrm{M}}\bar{\mathrm{F}}}, w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\bar{\mathrm{M}}\mathrm{F}}, w_{\mathrm{MF}}\}\rangle \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\varnothing & \varnothing & \{w_{\mathrm{M}\bar{\mathrm{F}}}\} & \{w_{\mathrm{M}\bar{\mathrm{F}}}\} & \{w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\mathrm{MF}}\}
\end{array}$$

From here, removing redundancy:

(15) $\langle \varnothing, \{w_{\mathrm{M}\bar{\mathrm{F}}}\}, \{w_{\mathrm{M}\bar{\mathrm{F}}}, w_{\mathrm{MF}}\}\rangle$

At this point, we evaluate the consequent at the updated path. *Frida passed* is false at (15), hence the conditional is false at (11). Notice the key point that guarantees the validity of CEM: nonmodal consequents are always evaluated at a single world.

Discussion. Before moving on to consequence, let me notice a feature of the semantics. Path semantics bears an obvious resemblance to Veltman's data semantics ([20]), since it tracks possible trajectories along which an information state might evolve. At the same time, just conditionals are treated very differently. Data semantics uses a strict conditional analysis. In path semantics, conditionals have no quantificational force of their own. The *if*-clause is used to update the path; the consequent is evaluated at the updated path as any other sentence. Strictly speaking, then, path semantics (somewhat similarly to the restrictor account in [12]) doesn't see conditionals as semantic entities at all. Conditionals are just ordinary sentences prefaced by a path-shifting device, i.e. the *if*-clause.

5 Logical consequence

5.1 Defining consequence

Path semantics allows us to define several notions of consequence. One captures preservation of truth at a path.

> **Path consequence.**
> $\phi_1, \ldots, \phi_n \vDash_{\mathrm{P}} \psi$ iff for all paths P such that $[\![\phi_1]\!]^P = 1, \ldots, [\![\phi_n]\!]^P = 1$, $[\![\psi]\!]^P = 1$

While path consequence is useful for several purposes, it is not the notion that best captures what follows from a set of accepted premises.[13] But a notion of this sort can be easily defined as follows.

> **Path-Informational consequence.**
> $\phi_1, ..., \phi_n \vDash_{PI} \psi$ iff, for all i such that $\phi_1, ..., \phi_n$ are true throughout i, ψ is true throughout i.

Path-Informational consequence is the analog, in the current framework, of Veltman's ([21]) test-to-test validity, or Yalcin's ([22]) informational consequence. Informally, it tracks what follows from an information state that validates certain premises. It is the obvious notion of consequence for assessing consistency and validity for asserted claims in natural language.

Path-Informational consequence vindicates both CEM and IMC, while blocking the collapse of *might*-conditionals onto bare conditionals.[14]

> **Fact 1.** $\vDash_{PI} (\phi > \psi) \vee (\phi > \neg\psi)$
>
> **Fact 2.** $(\phi > \neg\psi) \wedge (\phi > \Diamond\psi) \vDash_{PI} \bot$
>
> **Fact 3.** $\phi > \Diamond\psi \nvDash_{PI} \phi > \psi$

Notice also that, despite the validity of IMC, Duality fails.

> **Duality.** $\nvDash_{PI} (\phi > \Diamond\psi) \leftrightarrow (\neg(\phi > \neg\psi))$

6 Conclusion

Path semantics reconciles CEM and the inconsistency of $\phi > \neg\psi$ and $\phi > \Diamond\psi$, solving a problem that in various forms has been discussed since the beginning of modern work on conditionals. My proposal is confined to epistemic conditionals, but the puzzle generalizes to counterfactuals. The results of this paper encourage exploring the prospects for a general semantic framework for conditionals that accommodates both epistemic conditionals and counterfactuals.

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[13] To see this, notice that path consequence fails to vindicate one of the signature inference patterns of the semantics in [21] and [22], i.e. what Yalcin calls 'Łukasiewicz's principle'.

> **Łukasiewicz's principle.** $\neg\phi \vDash \neg\Diamond\phi$

[14] Here are some intuitive proofs. As for CEM: $(\phi > \psi) \vee (\phi > \neg\psi)$ is true at all paths, hence it is true throughout any i. As for IMC: if $\phi > \psi$ is true at all paths terminating at i, then i contains no $\phi \wedge \neg\psi$-worlds, hence $\phi > \Diamond\neg\psi$ is false.

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